Annex 3.2

Validators Report : Theoretical analysis of the algorithms

14-April-08

written by:

Alain Billionnet

Ingo Ellersdorfer

Sourour Elloumi

Bernard Fortz

1 The CWE Project and the task of the validators

The CWE project aims at coupling the 4 day-ahead markets of the CWE power exchanges, through a flow-based implicit capacity allocation mechanism. The core of this mechanism will be an algorithm calculating the optimised market results (volumes and prices), from an input consisting of information from the PXs' order books and network parameters provided by the TSOs.

The CWE project has set up a workstream (the "Algorithm Design Workstream"), with the mission of designing the coupling algorithm. The main task of the Validators was to judge on the selection work done by the Algorithm Design Workstream, and to compare the proposed algorithms, both theoretically and based on the results of empirical testing performed by the candidates. This comparison also provides a list of advantages and disadvantages of each algorithm. The validators have to provide together one common report, comprising three parts :

Part I : Evaluation criteria and testing methodology.

Part II : Theoretical analysis of the algorithms.

Part III : Testing results for selection including their final recommendations.

The present report has been written by the validators after understanding the functioning of the two candidate algorithms COSMOS and MLC. It concerns the theoretical comparison of these two algorithms from the following points of view: mandatory requirements, quality of the solution, simplicity, performance, reliability and extendibility.

2 Short description of the algorithms

The building block for both algorithms is a mathematical model for the block free problem. This model intends to maximize total net utility. As the capacity for cross-market exchanges is limited, congestion must be taken into account (ATC or flow-based formulation). This might lead to different clearing prices in different markets. When all orders can be partially filled (block free), the problem is well solved and the solution is unique, because the model is a concave quadratic or linear optimization problem. Moreover, the property that *in-the-money* orders are accepted, and *out-of-money* orders are rejected holds as a direct consequence of duality theory.

Unfortunately, the markets under consideration also allow block orders that are *"fill-or-kill"*, i.e. they must be completely executed or completely rejected. This makes the problem a combinatorial optimisation problem, which might be infeasible if both in-the-money block orders must be accepted and out-of-the money block orders must be rejected. To make the problem feasible, in-the-money block orders are allowed to be paradoxically rejected, since paradoxically accepted orders are forbidden by market rules.

The main difference between the two proposed approaches is the way they deal with that combinatorial aspect.

2.1 MLC

MLC uses a heuristic approach for tackling the combinatorial aspect, while explicitly trying to keep the difference between the market price and the rejected block prices (called Delta P) under control. As an expected consequence, the number of paradoxically rejected blocks (PRBs) is expected to remain small.

The whole system is based on independent block selector modules (one for each market). These modules select a set of winning blocks, leading to new Net Block Volumes (NVB). These new volumes are taken into account by translating Net Export Curves (NEC) that are sent to the central module in order to solve the mathematical model. The solver sends back market clearing prices to the block selection modules. To avoid infeasibilities, a negative limit price far below the low price boundary is assigned to sale blocks and a positive limit price far beyond the high price boundary is assigned to buy blocks.

The selection of blocks is made in such a way that the cumulated delta P value is upper bounded by some parameter DP. During the process, DP is increased to allow more PRBs to appear in the solution. At each step, the set of winning blocks is reduced by removing blocks which were not accepted in the previous iteration but accepted in the current iteration. These blocks become PRBs. The procedure iterates until the cumulated delta P of PRBs becomes lower than DP.

The heuristic starts with all blocks accepted (DP = 0), then the block selection module iteratively modifies the set of winning blocks. If this procedure fails to find a feasible solution, DP is increased to allow for more paradoxically rejected blocks. These rules ensure that the set of winning blocks is monotonically decreasing, so that the procedure converges.

To increase the quality of the solutions, a block reintegration procedure is also implemented that allow previously rejected blocks to be reintegrated if this improves the quality of the solution. This procedure starts with testing of the block with the highest deltaP. This prevents premature convergence of the algorithm.

The procedure stops when the set of winning blocks does not change anymore – which also means that the MCP of each market remains the same as in the previous iteration -, and the algorithm is tuned such that DP leads to a stable solution quickly (well before the time limit of 10 minutes is reached).

2.2 COSMOS

Cosmos is based on an exact method for tackling the combinatorial aspect. The objective function is the net utility value. The mathematical model, including the set of integer variables due to block orders, is submitted to a “classical” branch and bound process, where integrality of the block variables is relaxed (meaning that some block orders could be partially accepted at each node). In order to find a feasible solution quickly, branching is first done on the “kill” direction, and a depth-first-search strategy is applied. When an integral solution is found, clearing prices compatible with high-level properties are computed, if they exist. If such prices do not exist, the set of accepted blocks is infeasible and is forbidden by adding a new constraint to the model. It has been mathematically proved by the COSMOS developers that these new constraints ensure feasibility and optimality of the final solution.

The algorithm stops either when an optimal solution has been (provably) obtained, or when the time limit is reached. In this case, the best feasible solution found is returned, as well as the dual bound provided by the optimizer.

3 Comparison of the algorithms

3.1 Mandatory requirements

According to the documentation received and the forms filled by the candidates, both approaches fulfil the mandatory and most of the additional requirements described in document “DES-3: Algorithms requirements”.

In particular, all mandatory order types are correctly taken into account, and congestion control (flow-based or ATC) is also implemented in both approaches.

High Level Properties (HLP) – relating market clearing prices and congestion shadow prices - are also satisfied in both approaches, except for price boundaries.

If price boundaries are imposed, prices satisfying all HLP might not exist. None of the algorithms handles price boundaries. Negative price boundaries are just a special case of price boundaries. But in addition to making the HLP possibly infeasible, they might also lead to negative clearing prices.

At the moment, we think that the algorithms SHOULD NOT take price boundaries into account, because it is theoretically impossible to make them compatible with HLP. This does not prevent markets to impose price boundaries on the participants, but the final clearing prices should not be constrained within the boundaries. Another approach would be to redefine the HLP to make them consistent with the price boundaries, but this is definitely out of the scope of the analysis of the algorithms.

3.2 Optimality and quality of the solution

3.2.1 Optimality of the solution

Due to the methodologies chosen, COSMOS is the only algorithm guaranteed to converge to an optimal solution (if ran for long enough), or to provide a measure of the quality of the solution in case the time limit is reached without proving optimality. If no PRBs are present in the optimal solution, COSMOS will find it without branching.

This is not true for MLC: in the case where there exists a solution with no PRBs, MLC will find it only if the market clearing price and the set of winning blocks converge to that solution before DP is increased. Nothing ensures this will be the case, and as soon as DP is increased, PRBs might appear due to the block selection process even if a solution without PRBs exists.

3.2.2 Quality of the solution in terms of market aspects

MLC explicitly manages market aspects (number of PRBs, DeltaP) by controlling an upper bound DP on the cumulative DeltaP. Roughly speaking, the heuristic tries to find the smallest DP value for which a feasible solution exists, by selecting accepted blocks with a cumulative DeltaP less than DP (with respect to the previous iteration). Then it maximizes welfare over this set of winning blocks, which is monotonically decreasing (while DP is increasing) in order to ensure convergence.

In COSMOS, optimization is done on the total net utility only. The assumption is done that small DeltaP and number of PRBs will come as a consequence of this optimization.

The weighting between these aspects still remains open. The tuning of the algorithms might depend on this decision, and the testing results might differ significantly following this tuning. If the only objective is maximizing welfare, COSMOS seems the most elegant approach on a theoretical point of view, but only empirical results can tell if the method remains competitive if market aspects are integrated.

In our understanding of the mathematical model used in COSMOS, it would not be possible to simply integrate market aspects in the objective function, as no prices explicitly appear in the model. However, the assumption behind the algorithm is that market aspects are strongly related with welfare. If empirical results show the converse, the authors propose to handle these market aspects heuristically, by keeping the best solution (with respect to the market aspects) found in the branch-and-bound tree. Doing that, the algorithm would no longer guarantee optimality of the solution from the net utility point of view.

3.3 Simplicity

MLC is made of two modules: a heuristic block selection and the optimization of the mathematical model. Block selection is based on simple rules that are easy to implement, and the mathematical model can be implemented in any QP solver on the market.

COSMOS is based on a specific implementation of a branch-and-bound strategy with addition of cutting planes. This can also be implemented using any QP solver on the market (through the solver's callable library or through a modelling language).

We can conclude that both approaches meet this criterion.

3.4 Performance

Ability to provide a feasible solution within the time limit:

COSMOS uses a special branching rule that first kills block orders. This approach will eventually lead to a feasible solution. In the worst case, as many QPs as block orders could be solved before a first feasible solution is found, but in practice this number should be much smaller and this should be performed within the given time limit. This solution is trivial.

For MLC, a feasible solution is found when DP becomes large enough. The rate of convergence will highly depend on the evolution of the β parameters, and on the difficulty of the instance. Based on the parameters setting proposed, convergence is forced after 70 iterations (β=1).

3.5 Reliability

Both algorithms are based on the solution of a large number of convex quadratic programs. Many efficient algorithms are known for this problem which is the heart of both methods. COSMOS and MLC use the quadratic solver of CPLEX, a commercial and proven robust optimizer. Moreover, both algorithms use similar pre-processing techniques to improve the speed of convergence of CPLEX in successive iterations.

3.6 Extendibility

Extension to a bigger number of markets is theoretically feasible for both methods, and should be tested for efficiency empirically. Based on the current documentation in our possession, COSMOS fully supports additional order types like linked block orders and flexible hourly orders. MLC has limited support for these orders (the “fill-or-kill” constraint is being relaxed).

Additional considerations that can be modelled as linear relations is straightforward in COSMOS while any such change would require some algorithmic adaptation in MLC block selection module. Nonlinear relations (although very unlikely) need to be dealt with heuristically in both algorithms. In MLC, these adaptations might also be incompatible with the “fill-or-kill” constraint.

But in practice every additional consideration has to be studied separately.

4 Conclusion

COSMOS is an exact method based on a branch-and-bound approach aimed at maximizing welfare. Other market aspects are implicitly considered based on the hypothesis that these market aspects are strongly correlated with welfare. Branching rules ensure a feasible solution is computed quickly, while the available computing time is used to improve the solution, and if possible, prove its optimality.

MLC is a heuristic-based approach trying to balance welfare and market aspects, with the aim of achieving fast convergence.

Both algorithms satisfy all mandatory requirements, except price boundaries. However, price boundaries are incompatible with other requirements. Therefore, it is impossible to solve this problem on the algorithmic side. New realistic (and feasible) high level properties should be defined if price boundaries are required.

On a theoretical basis, COSMOS seems the most promising approach, provided the experimental results confirm the hypothesis that market aspects are strongly correlated with welfare, and that the branch-and-bound approach scales well when problem size increases (in particular, it should always be able to find a good quality feasible solution even if optimality is not proven). If this is is not the case, MLC would be preferred as market aspects are explicitly taken into account and computing time is strictly under control.